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Mathematics - II

SYLLABUS

Unit I : Integral Calculus and Numerical Technique

Unit II : Integral Calculus

Unit III : Differential Equation and its Applications

Unit IV : Multiple Integration

Unit V : Vector Calculus

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Unit VI : Numerical Method - I

Unit VII : Numerical Method - II

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MIND MAP

INTEGRAL CALC AND NUMERICAL TECHNIQUE

Differential Equations (D.E.)

an eq^n involving dependent variables, independent variables and the differential coefficients of various orders.

SOLVING METHODS

- Exact D.E.
- Reducible to exact D.E.
- Linear differential eq^ns

Exact D.E.

for eq^n $M\,dx + N\,dy = 0$;

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

→ x is const.

⇒ $G.S. = \int M\,dx + \int [\text{Terms of } N \text{ not containg } x]\,dy = c$

→ y is const.

$G.S. = \int N\,dy + \int [\text{Terms of } N \text{ not containing } y]\,dx = c$

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Reducible to Exact D.E.

1. for $M\,dx + N\,dy = 0$, if it is homogeneous, to make it exact, the I.F. will be :

$$I.F. = \frac{1}{Mx + Ny}$$

if $Mx + Ny \neq 0$

2. For $M f_1(xy)\,dx + N f_2(xy)\,dy = 0$, to make it exact,

$M = M f_1(xy)$

$N = N f_2(xy)$

$$I.F. = \frac{1}{Mx - Ny}$$

if $Mx - Ny \neq 0$

3. Otherwise,

Cal. $f(x) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$

$$\Rightarrow I.F. = e^{\int f(x)\,dx}$$

4. $f(y) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M}$

$$\Rightarrow I.F. = e^{\int f(y)\,dy}$$

Checking order:
- → differentiate
- → homog.
- → $My\,dx + Nx\,dy = 0$
- → others
- → $\dfrac{dx}{dy} + px = Q$

OR

$\dfrac{dy}{dx} + py = Q$

Linear Differential Eq^{n}s

For eq^n $\left[\dfrac{dx}{dy} + px = Q\right]$,

→ $I.F. = e^{\int p\,dy}$

→ G.S.: $x(I.F.) = \int Q\,(I.F.)\,dy + c$

For eq^n $\left[\dfrac{dy}{dx} + py = Q\right]$,

→ $I.F. = e^{\int p\,dx}$

→ G.S.: $y(I.F.) = \int Q\,(I.F.)\,dx + c$

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Differential Equations and Its Applications

An eq^n involving dependent variables, independent variables and the differential coefficients of various orders.

Any eq^n which contains a derivative is a differential

Diff. eq^n

ODE | PDE

Ordinary Differential Eq^n (ODE)

- ~~only~~ involves only one independent variable and one dependent variable
- their derivatives with respect to independent variable.

Partial Differential Eq^n (PDE)

- involves multiple independent variables.
- one dependent variable.
- thier partial derivative with respect to independent. variables.

Sol^n of diff eq^{n}s / Gen sol^n of diff eq^{n}s (G.S.)

A relation between the dependent and independent variable which is free from derivatives which is free from derivatives which satisfy the given diff. eq and which contains arbitrary constants equal to order of the

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differential eq^{ns}.

- The G.S. of D.E. of order n must have n arbitrary constants

- Suppose D.E. $\frac{dy}{dx} = \cos x$ $\left[\int dy = \int \cos x \, dx\right]$

its sol^n is $y = \sin x + c$ $\left[y = \sin x + c \right]$

Differential of first order and first degree

D.E. of first order and first degree is of the form $M(x,y)dx + N(x,y)dy = 0$

OR

$$\frac{dy}{dx} = f(x,y)$$

- Methods to solve (s) SharkCoders
  - → Variable seperable form
  - → reducible to variable seperable form
  - → homogeneous D.E.
  - → non-homogeneous D.E. / reducible to homogeneous eq^n
  - → exact D.E.
  - → reducible to exact D.E.
  - → linear differential eq^{n}s
  - → reducible to linear diff.

Exact D.E.::

- the D.E. of type $M(x,y)dx + N(x,y)dy = 0$ is said to be exact if there exists some func^n 'u' such that $M(x,y)dx + N(x,y)dy = du$
- necessary and sufficient condition that $M(x,y)dx + N(x,y)dy = 0$ be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

~~when the condition~~ of exactness ~~is satisfied,~~ the
~~G.S. can be obtained by the following~~ rote

• Rule I: (y = const)

G.S. $\int M\,dx + \int [\text{Terms of N not containing } x]\,dy = c$

• Rule II: (x = const)

G.S. $\int N\,dy + \int [\text{Terms of M not containing } y]\,dx = c$

Q. Solve the D.E.,

$(x^2+y)dx + (y^2+x)dy = 0$

Ans. Comparing with $Mdx + Ndy = 0$

where $M = x^2+y$, $N = y^2+x$

$\frac{\partial M}{\partial y} = 1$, $\frac{\partial N}{\partial x} = 1$

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$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ Given D.E. is exact

G.S. $\int_{y=\text{const}} M\,dx + \int [\text{Terms of N not containing } x]\,dy = c$

$\Rightarrow \int_{y=\text{const}} (x^2+y)dx + \int y^2\,dy = c$

$\therefore \frac{x^3}{3} + yx + \frac{y^3}{3} = c$

$\therefore x^3 + 3xy + y^3 = 3c = c_1$

Q. Solve the D.E.,

$$(x+y-2)\,dx + (x-y+4)\,dy = 0$$

Ans. Comparing with $M\,dx + N\,dy = 0$

where $M = x+y-2$, $N = x-y+4$

$$\frac{\partial M}{\partial y} = 1 \quad , \quad \frac{\partial N}{\partial x} = 1$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

~~Given~~ $\Rightarrow$ D.E. is exact.

$\Rightarrow$ G.S. $\int\limits_{y=\text{const}} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$

$$\therefore \int (x+y-2)\,dx + \int (-y+4)\,dy = c$$

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$$\frac{x^2}{2} + yx - 2x + \frac{-y^2}{2} + 4y = c$$

$$x^2 + 2xy - 4x - y^2 + 8y = 2c = c_1$$

Q. Solve the D.E.,

$$y\,dx = (\sin y - x)\,dy$$

Ans. $y\,dx - (\sin y - x)\,dy = 0$

where $M = y$, $N = -(\sin y - x)$

$$\frac{\partial M}{\partial y} = 1 \quad , \quad \frac{\partial N}{\partial x} = 1$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 1$$

$\Rightarrow$ D.E. is exact

$$G.S. \int_{y=const} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\Rightarrow \int y\,dx + \int (-\sin y)\,dy = c$$

$$\therefore \quad xy + \cos y = c$$

Q. $\dfrac{dy}{dx} = \dfrac{2x-3y+1}{3x+4y-5}$

Ans. $(2x-3y+1)\,dx = (3x+4y-5)\,dy$

$\int(2x-3y+1)\,dx = \int(3x+4y-5)\,dy$

$\cancel{\frac{2x^2}{2}} \; \cancel{-3xy+x} = \cancel{3xy + \frac{4y^2}{2}} \; \cancel{-5y}$

Comparing $M\,dx + N\,dy = 0$

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$M = 2x - 3y + 1$, $N = -3x - 4y + 5$

$\dfrac{\partial M}{\partial y} = -3$, $\dfrac{\partial N}{dx} = -3$

$$\therefore \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = -3$$

$\Rightarrow$ D.E. is exact.

$$\therefore \; G.S. \int_{y=const} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$= \int(2x-3y+1)\,dx + \int(-4y+5)\,dy = c$$

$$= x^2 - 3xy + x - 2y^2 + 5y = c$$

Q. Solve the D.E.

$$\frac{dy}{dx} = \frac{1 + y^2 + 3x^2y}{1 - 2xy - x^3}$$

Ans. $(1 + y^2 + 3x^2y)\,dx - (1 - 2xy - x^3)\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$\therefore M = 1 + y^2 + 3x^2y$, $N = -1 + 2xy + x^3$

$$\frac{\partial M}{\partial y} = 2y + 3x^2 \qquad \frac{\partial N}{\partial x} = 2y + 3x^2$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is exact.

G.S. $\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = C$

$$= \int (1 + y^2 + 3x^2y)\,dx + \int (-1)\,dy = c$$

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$$= x + xy^2 + x^3y - y = C$$

Q. Solve the D.E.,

$$(y^2 e^{xy^2} + 4x^3)\,dx + (2xy\,e^{xy^2} - 3y^2)\,dy = 0$$

Ans. Comparing with $M\,dx + N\,dy = 0$

$M = y^2 e^{xy^2} + 4x^3$, $N = 2xy\,e^{xy^2} - 3y^2$

$$\frac{\partial M}{\partial y} = y^2 (e^{xy^2})(2xy) + e^{xy^2} \cdot 2y \qquad \frac{\partial N}{\partial x} = (2xy)(e^{xy^2})(y^2) + e^{xy^2}(2y)$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Longrightarrow$ D.E. is exact

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int (y^2 e^{xy^2} + 4x^3)\,dx + \int (-3y^2)\,dy = c,$$

$$y^2 \frac{e^{xy^2}}{y^2} + \frac{4x^4}{4} - \frac{3y^3}{3} = c$$

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$\therefore y^4 - x^2y^2 + x^4 - y^8 = c$

4) $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Ans. Comparing with $M\,dx + N\,dy = 0$

$M = x^2y - 2xy^2$

$N = 3x^2y - x^3$

$\frac{\partial M}{\partial y} = x^2 - 4xy \qquad \frac{\partial N}{\partial x} = 6xy - 3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ The D.E. is not exact and is homogeneous.

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$\text{I.F.} = \frac{1}{Mx + Ny}$

$Mx + Ny =$

$(x^2y - 2xy^2)x + (3x^2y - x^3)y =$

$x^3y - 2x^2y^2 + 3x^2y^2 - x^3y =$

$3x^2y^2 - 2x^2y^2 = x^2y^2$

$\therefore \text{I.F.} = \frac{1}{x^2y^2}$

$\Rightarrow \frac{1}{x^2y^2}(x^2y - 2xy^2)\,dx - \frac{1}{x^2y^2}(x^3 - 3x^2y)\,dy = 0$

$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\left(\frac{3}{y} - \frac{x}{y^2}\right)dy = 0$

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$$-2\log x + 3\log y + \frac{x}{y} = c$$

Integrating factor

A func^n $\lambda(x,y)$ is said to be an integrating factor of the eq^n $M\,dx + N\,dy = 0$, if it is possible to obtain a func^n $u(x,y)$ such that $\lambda(M\,dx + N\,dy) = du$

OR

An integrating factor is a multiplying factor by which the eq^n can be made exact.

- Rules 1:

If $Mx + Ny \neq 0$ and given D.E. is homogeneous,

$$\text{I.F.} = \frac{1}{Mx + Ny}$$

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Q. Solve the following D.E.,

$$(3xy^2 - y^3)\,dx + (xy^2 - 2x^2y)\,dy = 0$$

Ans Comparing with $M\,dx + N\,dy = 0$,

$$M = 3xy^2 - y^3$$
$$N = xy^2 - 2x^2y$$

$$\frac{\partial M}{\partial y} = 6xy - 3y^2 \qquad \frac{\partial N}{\partial x} = y^2 - 4xy$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact but it is homogeneous.

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$$\text{I.F.} = \frac{1}{Mx + Ny}$$

$$Mx + Ny = (3xy^2 - y^3)x + (xy^2 - 2x^2y)y$$
$$= 3x^2y^2 - xy^3 + xy^3 - 2x^2y^2$$
$$= 3x^2y^2 - 2x^2y^2$$
$$= x^2y^2 \neq 0$$

$$\therefore \text{I.F.} = \frac{1}{x^2y^2}$$

$$\frac{1}{x^2y^2}(3xy^2 - y^3)dx + \frac{1}{x^2y^2}(xy^2 - 2x^2y)dy =$$

$$\left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \left(\frac{1}{x} - \frac{2}{y}\right)dy =$$

G.S. is :-

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int\left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \int\left(-\frac{2}{y}\right)dy = c$$

$$3\log x - \frac{y}{x} + 2\log y = c$$

$$\therefore 3\log x - 2\log y - \frac{y}{x} = c$$

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Q. $(x^2-3xy+2y^2)dx + [x(3x-2y)]dy = 0$

Ans. $M = x^2 + 2y^2 - 3xy$

$N = 3x^2 - 2xy$

$\frac{\partial M}{\partial y} = 4y - 3x$ $\qquad \frac{\partial N}{\partial x} = 6x - 2y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ D.E. is not exact but it is homogeneous.

~~Mdx + Ndy = 0~~

$$\text{I.F.} = \frac{1}{Mx+Ny}$$

$$Mx + Ny = (x^2+2y^2-3xy)x + (3x^2-2xy)y$$
$$= x^3 + 2xy^2 - 3x^2y + 3x^2y - 2xy^2$$
$$= x^3 \neq 0$$

$$\therefore \text{I.F.} = \frac{1}{Mx+Ny} = \frac{1}{x^3}$$

$$\frac{1}{x^3}(x^2+2y^2-3xy)dx + \frac{1}{x^3}(3x^2-2xy)dy = 0$$

$$\left(\frac{1}{x} + \frac{2y^2}{x^3} - \frac{3y}{x^2}\right)dx + \left(\frac{3}{x} - \frac{2y}{x^2}\right)dy = 0$$

Comparing with $M\,dx + N\,dy = 0$

$M = \left(\frac{1}{x} + \frac{2y^2}{x^3} - \frac{3y}{x^2}\right)$

$N = \left(\frac{3}{x} - \frac{2y}{x^2}\right)$

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$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = 0$$

$$\int\left(\frac{1}{x} + \frac{2y^2}{x^3} - \frac{3y}{x^2}\right)dx + \int 0\,dy = 0$$

$$\log x - \frac{y^2}{x^2} + \frac{3y}{x} = 0$$

<u>H.W.</u>

Q. $(x^3+y^3)\,dx - xy^2\,dy = 0$

Ans. Comparing with $M\,dx + N\,dy = 0$,

$M = x^3 + y^3$

$N = -xy^2$

$$\frac{\partial M}{\partial y} = 3y^2 \qquad \frac{\partial N}{\partial x} = -y^2$$

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$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact ~~and~~ but it is homogeneous.

$$\text{I.F.} = \frac{1}{Mx+Ny}$$

$$Mx+Ny = (x^3+y^3)x + (-xy^2)y$$
$$= x^4 + xy^3 - xy^3$$
$$= x^4$$

$$\therefore \text{I.F.} = \frac{1}{x^4}$$

$$\frac{1}{x^4}(x^3+y^3) - \frac{1}{x^4}(-xy^2) = 0$$

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$$\left(\frac{1}{x} + \frac{y^3}{x^4}\right)dx + \left(-\frac{y^2}{x^3}\right)dy = 0$$

Comparing with ~~Mx + Nd~~ $Mx + Ny$ $Mdx + Ndy = 0$

$$M = \left(\frac{1}{x} + \frac{y^3}{x^4}\right)$$

$$N = \left(-\frac{y^2}{x^3}\right)$$

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = 0$$

$$\int \left(\frac{1}{x} + \frac{y^3}{x^4}\right)dx + \int 0\,dy = 0$$

$$\log x - \frac{y^3}{3x^3} = c$$

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$\therefore \cancel{\log x = \frac{y^3}{x^3}}$

Q. $(xy - 2y^2)dx - (x^2 - 3xy)\,dy = 0$

Ans. Comparing with $Mdx + Ndy = 0$,

$M = xy - 2y^2$

$N = 3xy - x^2$

$$\frac{\partial M}{\partial y} = x - 4y \qquad \frac{\partial N}{\partial x} = 3y - 2x$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

$\therefore$ D.E. is not exact but it is homogeneous.

$$\text{I.F.} = \frac{1}{Mx + Ny}$$

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$Mx + Ny = \cancel{0}\ C$

$(xy - 2y^2)x + (3xy - x^2)y = c$

$x^2y - 2xy^2 + 3xy^2 - x^2y = c$

$xy^2 \neq 0$

$\therefore \text{I.F.} = \dfrac{1}{xy^2}$

$$\frac{1}{xy^2}(xy - 2y^2)dx + \frac{1}{xy^2}(3xy - x^2)dy = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx + \left(\frac{3}{y} - \frac{x}{y^2}\right)dy = 0$$

Comparing with $Mdx + Ndy = 0$

$M = \left(\dfrac{1}{y} - \dfrac{2}{x}\right)$

$N = \left(\dfrac{3}{y} - \dfrac{x}{y^2}\right)$

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = 0$$

$$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\left(\frac{3}{y}\right)dy = 0$$

$$\therefore \frac{x}{y} - 2\log x + 3\log y = 0$$

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Rule II:

If $Mx - Ny \neq 0$ and the given D.E. has the form $f_1(x,y)\, y\, dx + f_2(x,y)\, x\, dy = 0$, then

$$I.F. = \frac{1}{Mx - Ny}$$

Q. Solve the following D.E.,

$(1+xy)y\,dx + (1-xy)x\,dy = 0$

Ans. Given D.E. is $(1+xy)y\,dx + (1-xy)x\,dy$ ——— (1)

Comparing with $Mx + Ny = 0$

$M = (1+xy)y = y + xy^2$

$N = (1-xy)x = x - x^2 y$

$$\frac{\partial M}{\partial y} = 1 + 2xy \qquad \frac{\partial N}{\partial x} = 1 - 2xy$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact but D.E. is of the form $f_1(x,y)\, y\, dx + f_2(x,y)\, x\, dy = 0$

Also $Mx - Ny =$

$(1+xy)yx - (1-xy)xy =$

$xy + x^2y^2 - xy + x^2y^2 =$

$2x^2y^2$

$$I.F. = \frac{1}{2x^2y^2}$$

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Multiply ① by I.F.,

$$\frac{1}{2x^2y^2}(1+xy)y\,dx + \frac{1}{2x^2y^2}(x-x^2y)\,dy = 0$$

$$\left(\frac{1}{2x^2y^2} + \frac{1}{2xy}\right)y\,dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right)dy = 0$$

$$\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right)dy = 0 \quad \text{②}$$

② is exact, G.S. is :

$$\int_{y=\text{const}} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

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$$\int\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \int\left(\frac{-1}{2y}\right)dy = c$$

$$\frac{-1}{2xy} + \frac{\log x}{2} - \frac{\log y}{2} = c$$

$$\frac{1}{2}\left(\frac{-1}{xy} + \log x - \log y\right) = c$$

$$\log\left(\frac{x}{y}\right) - \frac{1}{xy} = 2c = c_1$$

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1) $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$

Ans. $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0$ ——— (1)

Comparing with $Mx + Ny = 0$

$M = (xy^2 + 2x^2y^3)$

$N = (x^2y - x^3y^2)$

$\frac{\partial M}{\partial y} = 2xy + 6x^2y^2$ $\quad$ $\frac{\partial N}{\partial x} = 2xy - 3x^2y^2$

$\therefore$ D.E. is not exact but it is of the form

$f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy = 0$.

$Mx - Ny =$

$(xy^2 + 2x^2y^3)x - (x^2y - x^3y^2)y =$

$x^2y^2 + 2x^3y^3 - x^2y^2 + x^3y^3 =$

$3x^3y^3$.

$\therefore$ I.F. $= \frac{1}{3x^3y^3}$

Multiplying (1) with I.F.

$\frac{1}{3x^3y^3}(xy^2 + 2x^2y^3)\,dx + \frac{1}{3x^3y^3}(x^2y - x^3y^2)\,dy = 0$

$\left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = 0$

$\frac{\cancel{1}}{3x}$

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$$\int_{y=const} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int \left(\frac{1}{3x^2y} + \frac{2}{3x}\right) dx + \int \left(\frac{-1}{3y}\right) dy = c$$

$$\frac{-1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = c$$

$$\frac{2\log x - \log y}{3} - \frac{1}{3xy} = c$$

$$2\log x - \log y - \frac{1}{xy} = 3c = C_1$$

$$\log \frac{x^2}{y} - \frac{1}{xy} = C_1$$

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Q. $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

Ans. $(x^2y^2 + xy + 1)y\,dx + (x^2y^2 - xy + 1)x\,dy = 0$

$(x^2y^3 + xy^2 + y)\,dx + (x^3y^2 - x^2y + x)\,dy = 0$ —— (1)

Comparing with ~~Mdx + Ndy = 0~~ $M\,dx + N\,dy = 0$

$M = (x^2y^3 + xy^2 + y)$

$N = (x^3y^2 - x^2y + x)$

$$\frac{\partial M}{\partial y} = 3x^2y^2 + 2xy + 1 \qquad \frac{\partial N}{\partial x} = 3x^2y^2 - 2xy + 1$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact and it is of the form

$f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy = 0$

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$Mx - Ny =$

$(x^2y^3 + xy^2 + y)x - (x^3y^2 - x^2y + x)y =$

$x^3y^3 + x^2y^2 + xy - x^3y^3 - x^2y^2 + xy =$

$2xy \neq 0$

$\text{I.F.} = \dfrac{1}{2xy}$

Multiplying ① with I.F.,

$$\frac{1}{2xy}(x^2y^3 + xy^2 + y)\,dx + \frac{1}{2xy}(x^3y^2 - x^2y + x)\,dy = 0$$

$$\left(\frac{xy^2}{2} + \frac{y}{2} + \frac{1}{2x}\right)dx + \left(\frac{x^2y}{2} - \frac{x}{2} + \frac{1}{2y}\right)dy = 0$$

Comparing with ~~$Mdx + Ndy = 0$~~ $Mdx + Ndy = 0$

$$M = \left(\frac{xy^2}{2} + \frac{y^2}{2} + \frac{1}{2x}\right)$$

$$N = \left(\frac{x^2y}{2} - \frac{x}{2} + \frac{1}{2y}\right)$$

$$\int_{y = \text{const}} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int \left(\frac{xy^2}{2} + \frac{y^2}{2} + \frac{1}{2x}\right)dx + \int \left(\frac{1}{2y}\right)dy = c$$

$$\frac{x^2y^2}{4} + \frac{2xy^2}{2} + \frac{\log x}{2} + \frac{\log y}{2} = c$$

$$x^2y^2 + 2xy^2 + 2\log x + 2\log y = 4c = C_1$$

$$x^2y^2 + 2xy^2 + \log x^2 + \log y^2 = 4c = C_1$$

$$x^2y^2 + 2xy^2 + \log\left(\frac{x^2}{y^2}\right) = c_1$$

SharkCoders

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Q. $y(xy-3)\,dx + x(3xy-3)\,dy = 0$

Ans. $(xy^2-3y)\,dx + (3x^2y-3x)\,dy = 0$ ——— (1)

Comparing with $M\,dx + N\,dy = 0$

$M = (xy^2-3y)$

$N = (3x^2y-3x)$

$\frac{\partial M}{\partial y} = 2xy - 3$ $\qquad$ $\frac{\partial N}{\partial x} = 6xy - 3$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial y}$

$\therefore$ D.E. is not exact but it is of the form

$f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy = 0$

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$\text{I.F.} = \frac{1}{Mx - Ny}$.

$Mx - Ny =$

$(xy^2-3y)x - (3x^2y-3x)y =$

$x^2y^2 - 3xy - 3x^2y^2 + 3xy =$

$-2x^2y^2$

$\text{I.F.} = \frac{-1}{2x^2y^2}$

Multiplying (1) with I.F.,

$\frac{-1}{2x^2y^2}(x^2y^2-3y)\,dx - \frac{1}{2x^2y^2}(3x^2y-3x) = 0$

30/1/24.

$$-\frac{1}{2} + \frac{3}{2x^2y} - 3$$

Q. $(x^2y^2+2)y\,dx + (2-2x^2y^2)x\,dy = 0$

Ans. $(x^2y^2+2)y\,dx + (2-2x^2y^2)x\,dy = 0$ ——— (1)

Comparing with $M\,dx + N\,dy = 0$

$M = x^2y^3 + 2y$

$N = 2x - 2x^3y^2$.

$$\frac{\partial M}{\partial y} = 3x^2y^2 + 2y \qquad \frac{\partial N}{\partial x} = 2 - 6x^2y^2.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

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$\therefore$ D.E. is not exact but it is of the form

$f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy = 0$

$Mx - Ny =$

$(x^2y^3+2y)x - (2x-2x^3y^2)y =$

$x^3y^3 + 2xy - 2xy + 2x^3y^3 =$

$3x^3y^3 \neq 0$

$$\therefore \text{I.F.} = \frac{1}{3x^3y^3}$$

$$\frac{1}{3x^3y^3}(x^2y^3+2y)\,dx + \frac{1}{3x^3y^3}(2x-2x^3y^2)\,dy = 0$$

$$\left(\frac{1}{3x} + \frac{2}{3x^3y^2}\right)dx + \left(\frac{2}{3x^2y^3} - \frac{2}{3y}\right)dy = 0$$

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~~$\frac{1}{3x}$~~

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = C$$

$$\int\left(\frac{1}{3x} + \frac{2}{3x^2y^2}\right)dx + \int \frac{2}{3y}\,dy = C$$

$\frac{x^{-2+1}}{-2}$

$$\frac{\log x}{3} - \frac{1}{3x^2y^2} + \frac{2}{3}\log y = C$$

$$\log x + \log y^2 - \frac{1}{x^2y^2} = 3C = c_1$$

$$\log(xy^2) - \frac{1}{x^2y^2} = 3C = C_1$$

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~~Case~~

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Case III: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \phi(x)$, then or const

I.F. $= e^{\int \phi x \, dx}$

Case IV: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \phi(y)$, then I.F. $= e^{\int \phi(y)}$ or const

Q. Solve the following D.E.:

$(x^2+y^2+1)\,dx - 2xy\,dy = 0$

Ans. $(x^2+y^2+1)\,dx - 2xy\,dy = 0$ —— ①

Comparing with $M\,dx + N\,dy = 0$

$M = x^2 + y^2 + 1$

$N = -2xy$

$\dfrac{\partial M}{\partial y} = 2y \qquad \dfrac{\partial N}{\partial x} = -2y$

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$\therefore \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

$\Rightarrow$ D.E. is not exact

$\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x} = 2y + 2y = 4y$

$\because$ it is a multiple of N

$\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{4y}{-2xy} = \dfrac{-2}{x} = \phi(x)$

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$$I.F. = e^{\int \phi(x)\,dx}$$
$$= e^{\int -\frac{2}{x}\,dx}$$
$$= e^{-2\log x}$$
$$= e^{\log x^{-2}}$$
$$= x^{-2}$$
$$I.F. = \frac{1}{x^2}$$

Multiply eqn ① by $1/x^2$,

$$\frac{1}{x^2}(x^2+y^2+1)\,dx - \frac{1}{x^2}(2xy)\,dy = 0$$

$$\left(1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right)dx + \left(\frac{-2y}{x}\right)dy = 0 \quad \text{—— ②}$$

SharkCoders

∴ ② is exact.
Its G.S.,

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int\left(1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right)dx + \int 0\,dy = c$$

$$x - \frac{y^2}{x} - \frac{1}{x} + \cancel{0y} = c$$

$$x^2 - y^2 - 1 + \cancel{0xy} = xc = c_1$$

$$\therefore x^2 - y^2 - 1 = xc = c_1$$

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Q. Solve the following D.E.,

$(2x \log x - xy)\,dy + 2y\,dx = 0$

Ans. $(2x \log x - xy)\,dy + 2y\,dx$ —— ①

Comparing with $M dx + N dy = 0$,

$M = \cancel{x\log x - xy}\; 2y$

$N = \cancel{\log}\; 2x \log x - xy$

$\frac{\partial M}{\partial y} = 2 \qquad \frac{\partial N}{\partial x} = 2\log x + 2 - y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

∴ D.E. is not exact.

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$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2 - 2\log x - 2 + y$

$= y - 2\log x$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{y - 2\log x}{\cancel{2}x(2\log x - y)}$

$= \frac{-1}{x} = \phi(x)$

~~Multiply ① by I.F.,~~

$\cancel{\frac{1}{x}(2y)\,dx}$

$\text{I.F.} = e^{\int \phi(x)\,dx}$

$= e^{\log x}$

$= x^{-1}$

$= \frac{1}{x}$

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Multiply ① by I.F.,

$$\frac{1}{x}(2y)\,dx + \frac{1}{x}(2x\log x - xy)\,dy = 0$$

$$\left(\frac{2y}{x}\right)dx + (2\log x - y)\,dy = 0 \quad \text{--- ②}$$

$\therefore$ D.E. is exact.

Its G.S. is.

$$\int_{y=\text{const}} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = 0$$

$$\int \left(\frac{2y}{x}\right) dx + \int (-y)\,dy = 0$$

SharkCoders

$$2y\log x - \frac{y^2}{2} = c$$

$$4y\log x - y^2 = 2c = c_1$$

Q. $(2x + e^x \log y)\,y\,dx + e^x\,dy = 0$

Ans. ~~(2x2+(e^x))~~

$$(2xy + ye^x \log y)\,dx + e^x\,dy = 0$$

Comparing with $M\,dx + N\,dy = 0$.

$M = (2xy + ye^x \log y)$

$N = e^x$

$$\frac{\partial M}{\partial y} = 2x + e^x \cdot \log y + e^x \qquad \frac{\partial N}{\partial x} = e^x$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

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$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ D.E. is not exact.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x + e^x \log y + e^x - e^x$$

$$= 2x + e^x \log y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{-(2x + e^x \log y)}{(2xy + e^x y \log y)}$$

$$= \frac{-(2x + e^x \log y)}{y(2x + e^x \log y)}$$

$$= \frac{-1}{y} = \phi(x)$$

$$\text{I.F.} = e^{\int \phi(x)\, dx}$$

$$= e^{\int (\frac{-1}{y})\, dx}$$

$$= e^{\log y^{-1}}$$

$$= \frac{-1}{y}$$

~~Comparin~~

Multiplying ① by I.F.,

$$\frac{-1}{y}(2xy + ye^x \log y)\, dx + \frac{-1}{y}(e^x)\, dy = 0$$

$$(-2x - e^x \log y)\, dx - \left(\frac{e^x}{y}\right) dy = 0 \quad \text{—— ②}$$

$\therefore$ ② is exact

Its G.S. is,

SharkCoders

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$$\int_{y=const} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = C$$

$$\int (-2x - e^x \log y)\,dx + \int 0\,dy = c$$

$$+\frac{2x^2}{2} + e^x \log y = c$$

$$+x^2 + e^x \log y = C$$

Q. $y \log y\,dx + (x - \log y)\,dy = 0$

Ans. $y \log y\,dx + (x - \log y)\,dy = 0$

Comparing with $M\,dx + N\,dy = 0$

$M = y \log y$

$N = x - \log y$.

SharkCoders

$$\frac{\partial M}{\partial y} = \log y + \frac{y}{y} = \log y + 1$$

$$\frac{\partial N}{\partial x} = 1 + 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \log y + 1 - 1$$

$$= \log y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{-\log y}{y \log y} = \frac{-1}{y}$$

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$\int 1 \cdot \log x \, dx \log x$ $\quad$ $x \log x$

$$I.F. = e^{\int p(x)\,dx}$$
$$= e^{\int -\frac{1}{y} dx}$$
$$= e^{\log y^{-1}}$$
$$= \frac{1}{y}$$

Multiply ① with $1/y$,

$$\frac{1}{y}(y \log y)\,dx + \frac{1}{y}(x - \log y)\,dy = 0$$

$$(\log y)\,dx + \left(\frac{x}{y} - \frac{\log y}{y}\right) dy = 0$$

$\therefore$ D.E. is exact

Its G.S. is,

SharkCoders

$$\int M\,dx + \int [\text{Terms of N not containing } x]\,dy = c$$

$$\int \log y\,dx + \int \left(-\frac{\log y}{y}\right) dy = c$$

$$x \log y + \frac{y(\log y - 1)}{y} + \log y = c$$

$$x \log y + y \log y - y + \log y = c$$

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Q. $(y-2x^3)dx - x(1-xy)dy = 0$

Ans. $(y-2x^3)dx - (x-x^2y)dy = 0$

$(y-2x^3)dx + (x^2y-x)dy = 0$ —— (1)

Comparing with $Mdx + Ndy = 0$,

$M = (y-2x^3)$

$N = (x^2y - x)$

$\frac{\partial M}{\partial y} = 1 + 0 = 1$ $\qquad$ $\frac{\partial N}{\partial x} = 2xy - 1$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ D.E. is not exact

~~[illegible]~~,

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$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - 2xy + 1$$

$$= 2 - 2xy$$

$$= 2(1-xy)$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2(1-xy)}{-x(1-xy)}$$

$$= \frac{-2}{x} = \phi(x)$$

$$\therefore \text{I.F.} = e^{\int \phi(x)\,dx}$$

$$= e^{-2\log x}$$

$$= e^{\log x^{-2}}$$

$$= x^{-2}$$

$$= \frac{1}{x^2}$$

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Multiply (1) with I.F.,

$$\frac{1}{x^2}(y-2x^3)dx + \frac{1}{x^2}(x^2y-x)dy=0$$

$$\left(\frac{y}{x^2}-2x\right)dx + \left(y-\frac{1}{x}\right)dy=0$$

∴ D.E. is now exact.
Its G.S.: is,

$$\int M\,dx + \int [\text{Terms of } N \text{ not containg } x]\,dy = c$$

$$\int\left(\frac{y}{x^2}-2x\right)dx + \int y\,dy = c$$

SharkCoders

$$\frac{-y}{x} - \frac{2x^2}{2} + \frac{y^2}{2} = c$$

$$y^2 - 2x^2 - \frac{2y}{x} = 2c$$

$$xy^2 - 2x^3 - 2y = 2xc = c_1$$

$$xy^2 - 2x^3 - 2y = c_1$$

Q. $(x-y^2)dx + 2xy\,dy = 0$

Ans. $(x-y^2)dx + 2xy\,dy = 0$ —— (1)

Comparing with $Mdx + Ndy = c$

$M = x - y^2$

$N = 2xy$

$$\frac{\partial M}{\partial y} = -2y \qquad \frac{\partial N}{\partial x} = 2y$$

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$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ D.E. is not exact

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -2y - 2y$$

$$= -4y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-4y}{2xy}$$

$$= \frac{-2}{x} = \phi(x)$$

$$\therefore \text{I.F.} = e^{\int \phi(x)\,dx}$$

$$= e^{\int (-\frac{2}{x})\,dx}$$

$$= e^{-2\log x}$$

$$= e^{\log x^{-2}}$$

$$= \frac{1}{x^2}$$

$\cancel{\frac{1}{x^2}}$ Multiply ① with I.F.,

$$\frac{1}{x^2}(x - y^2)\,dx + \frac{1}{x^2}(2xy)\,dy = 0$$

$$\left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx + \left(\frac{2y}{x}\right)dy = 0$$

$$\int M\,dx + \int [\text{Terms of } C \text{ not containing } x]\,dy = 0$$

$$\int \left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx + \int 0\,dy = 0$$

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$$\log x + \frac{y^2}{x} + 0 = c$$

$$x\log x + y^2 = xc = c_1$$

Q. $(x^2+y^2+x)dx + (xy)dy = 0$

Ans. $(x^2+y^2+x)dx + (xy)dy = 0$ ①

Comparing with $Mdx + Ndy = 0$,

$M = x^2+y^2+x$

$N = xy$

$\therefore \frac{\partial M}{\partial y} = 2y$ $\qquad \frac{\partial N}{\partial x} = y$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

SharkCoders

$\therefore$ D.E. is not exact.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\cancel{2y}\, y}{\cancel{2y}\, xy} = \frac{1}{x} = \phi(x)$$

$\therefore$ I.F. $= e^{\int c\,dx}$ $\qquad e^{\int \phi(x)\,dx}$

$= e^{\int \frac{1}{2}dx}$ $\qquad e^{\int \frac{1}{x}dx}$

$= e^{\frac{x}{2}}$ $\qquad e^{\log x}$

$=$ $\qquad x$

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Multiply ① with I.F.,

$$e^{x/2}(x^2+y^2+x)\,dx + e^{x/2}(xy)\,dy = 0$$

$$(e^{x/2}x^2 + e^{x/2}y^2 + e^{x/2}x)\,dx + (e^{x/2}xy)\,dy = 0$$

$\therefore$ D.E. is exact.

Its G.S. is

$$\int (e^{x/2}x^2 + e^{x/2}y^2 + e^{x/2}x)\,dx + \int 0\,dy = c$$

$$\frac{e^{x/2}\cancel{x}^3}{3} + e^{x/2}y^2 + \frac{e^{x/2}x^2}{2} = c$$

$$\frac{2e^{x/2}\cancel{x}^3 + 6e^{x/2}y^2 + 3e^{x/2}x^2}{6} = c$$

$$2e^{x/2}\cancel{x}^3 + 6e^{x/2}y^2 + 3e^{x/2}x^2 = 6c = c_1$$

~~Q. $y(x^2y + e^x)\,dx - e^x\,dy = 0$~~

$$x(x^2+y^2+x)\,dx + x(xy)\,dy = c$$

$$(x^3 + xy^2 + x^2)\,dx + (x^2y)\,dy = c$$

$\therefore$ D.E. is exact

Its G.S. is,

$$\int (x^3 + xy^2 + x^2)\,dx + \int 0\,dy = c$$

$$\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} + 0 = c$$

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31/1/24

Q. $y(x^2y + e^x)dx - e^x dy = 0$

Ans $(x^2y^2 + e^x y)\,dx - e^x dy = 0$

Comparing with $Mdx + Ndy = 0$

$M = x^2y^2 + e^x y$

$N = -e^x$

$$\frac{\partial M}{\partial y} = 2x^2y + e^x \qquad \frac{\partial N}{\partial x} = -e^x$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact.

Uto.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x^2y + e^x + e^x$$

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$$= 2(x^2y + e^x)$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{2(x^2y + e^x)}{-y(x^2y + e^x)}$$

$$= \frac{-2}{y} = \phi(y)$$

$$\text{I.F.} = e^{\int \phi(y)\,dy}$$

$$= e^{\int \frac{-2}{y}dy}$$

$$= e^{-2\log y}$$

$$= e^{\log y^{-2}}$$

$$= \frac{+1}{y^2}$$

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$$\frac{1}{y^2}(x^2y^2+e^xy)dx - \frac{1}{y^2}(e^x)dy=0$$

$$\left(x^2+\frac{e^x}{y}\right)dx - \left(\frac{e^x}{y^2}\right)\cdot dy=0$$

$\therefore$ D.E. is exact.

Its G.S. is;

$$\int_{y=\text{const}} M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

$$\int\left(x^2+\frac{e^x}{y}\right)dx - \int 0\,dy = c$$

$$\frac{x^3}{3}+\frac{e^x}{y}=c$$

SharkCoders

Q. $\frac{dy}{dx}(x+2y^3) = y + 2x^3y^2$

$(x+2y^3)dy = (y+2x^3y^2)dx$

$(y+2x^3y^2)dx - (x+2y^3)dy=0$

Comparing with $Mdx+Ndy=0$,

$M = y+2x^3y^2$

$N = -(x+2y^3) = -x-2y^3$

$\therefore \frac{\partial M}{\partial y} = 1+4x^3y \qquad \frac{\partial N}{\partial x} = -1$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ D.E. is not exact.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 + 4x^3y + 1$$

$$= 2(1 + 2x^3y)$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} = \frac{-2(1+2x^3y)}{y(1+2x^3y)}.$$

$$= -\frac{2}{y} = \phi(y)$$

$$\text{I.F.} = e^{\int \phi(y)\,dy}$$

$$= e^{\int (-\frac{2}{y})\,dy}$$

$$= e^{-2\log y}$$

$$= e^{\log y^{-2}}$$

$$= \frac{1}{y^2}$$

SharkCoders

Multiply ① by $1/y^2$,

$$\frac{1}{y^2}(2x^3y^2 + y)\,dx - \frac{1}{y^2}(x + 2y^3)\,dy = c$$

$$\left(2x^3 + \frac{1}{y}\right)dx + \left(\frac{x}{y^2} - 2y\right)dy = c$$

$\therefore$ D.E is exact

Its G.S. is,

$$\int M\,dx + \int [\text{Terms of } N \text{ not containing } x]\,dy = c$$

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$$\int\left(2x^3+\frac{1}{y}\right)dx+\int(-2y)\,dy=c$$

$$\frac{2x^4}{4}+\log y-\frac{2y^2}{2}=c$$

$$\frac{x^4}{2}+\log y-y^2=c$$

$$x^4+2\log y-2y^2=2c=c_1$$

1/2/24

Linear Differential eq^n of first order

A diff. eq^n is said to be linear if dependent variable and its derivative appear only in the first degree.

: Linear diff. eq^n in x

- a D.E. of the form $\frac{dx}{dy}+px=Q$ where p and q are func^n of y or constants.

- Method of sol^n:

  I.F. $=e^{\int p\,dy}$

  Its G.S.: $x(\text{I.F.})=\int Q\,(\text{I.F.})\,dy+c$

Linear D.E. in y

- a D.E. of the form $\frac{dy}{dx}+py=Q$ where p and q are funct^{n}s of x or constants.

: Method of sol^n

I.F. $=e^{\int p\,dx}$

Its G.S.: $y(\text{I.F.})=\int Q\,(\text{I.F.})\,dx+c$

SharkCoders

Q. $(x^2+1)\frac{dy}{dx} + 4xy = \frac{1}{(x^2+1)^2}$

Ans. $\frac{dy}{dx} + \frac{4xy}{(x^2+1)} = \frac{1}{(x^2+1)^3}$

which is linear in y.

Here $p = \frac{4x}{x^2+1}$, $q = \frac{1}{(x^2+1)^3}$

$$
\begin{aligned}
\text{I.F.} &= e^{\int p\,dx} \\
&= e^{\int \frac{4x}{x^2+1}\,dx} \\
&= e^{2\int \frac{2x}{x^2+1}\,dx} \\
&= e^{2\log(x^2+1)} \\
&= e^{\log(x^2+1)^2} \\
&= (x^2+1)^2
\end{aligned}
$$

$\left[\because \int \frac{f'(x)}{f(x)}\,dx = \log(f(x)\right.$

SharkCoders

G.S.

$y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + c$

$$
\begin{aligned}
y(x^2+1)^2 &= \int \frac{(x^2+1)^2}{(x^2+1)^3}\,dx + c \\
&= \int \frac{1}{x^2+1}\,dx + c
\end{aligned}
$$

$y(x^2+1)^2 = \tan^{-1}x + c$

$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x$

Page

Q. $x\cos x \dfrac{dy}{dx} + (\cos x - x\sin x)y = 1$

$$\frac{dy}{dx} + \frac{(\cos x - x\sin x)y}{x\cos x} = \frac{1}{x\cos x}$$

which is linear in y.

$$\frac{dy}{dx} + py = Q.$$

$$p = \left(\frac{\cos x}{x\cos x} - \frac{x\sin x}{x\cos x}\right)y = \left(\frac{1}{x} - \tan x\right)$$

$$Q = \frac{1}{x\cos x}$$

$$\text{I.F.} = e^{\int p\,dx}$$
$$= e^{\int \frac{1}{x} - \tan x\,dx}$$
$$= e^{(\log x - \log \sec x)}$$
$$= e^{\log \frac{x}{\sec x}}$$
$$= \frac{x}{\sec x} = x\cos x$$

SharkCoders

G.S.

$$y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + c$$
$$= \int \frac{1}{x\cos x}(x\cos x)\,dx + c$$
$$= \int 1\,dx + c$$

$$y(x\cos x) = x + c$$
$$xy\cos x - x = c$$
$$x(y\cos x - 1) = c$$

Q. $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1$

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) = \frac{dy}{dx}$$

$$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$\frac{dy}{dx} + \left(\frac{1}{\sqrt{x}}\right)y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

Comparing with $\dfrac{dy}{dx} + Py = Q$

$$P = \frac{1}{\sqrt{x}}$$

$$Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

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$$\text{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{\sqrt{x}}\,dx}$$

$$= e^{2\sqrt{x}}$$

$$y(\text{I.F.}) = \int Q\,(\text{I.F.})\,dx + c$$

$$y\left(e^{2\sqrt{x}}\right) = \int \frac{e^{-2\sqrt{x}}}{\sqrt{x}}\left(e^{2\sqrt{x}}\right)dx + c$$

$$= \int \frac{1}{\sqrt{x}}\,dx + c$$

$$ye^{2\sqrt{x}} = 2\sqrt{x} + c$$

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Q. $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Ans. $(1+y^2)\dfrac{dx}{dy} = \tan^{-1}y - x$

~~dx~~

$$\cancel{(1+y^2) = (\tan^{-1}y - x)\frac{dy}{dx}}$$

$$\cancel{\frac{dy}{dx} = \frac{1+y^2}{\tan^{-1}y - x}}$$

$$\cancel{\frac{dy}{dx} = \frac{1}{\tan^{-1}y - x} + \frac{y^2}{\tan^{-1}y - x}}.$$

$$\frac{dx}{dy} + px = Q$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \cancel{\frac{\tan^{-1}y}{1+y^2}}\;\frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1}y}{1+y^2}$$

Comparing with $\dfrac{dx}{dy} + px = Q$.

$$p = \frac{1}{1+y^2} \qquad Q = \frac{\tan^{-1}y}{1+y^2}.$$

$$\text{I.F.} = e^{\int p\,dy}$$
$$= e^{\int \frac{1}{1+y^2}dy}$$
$$= e^{\tan^{-1}y}$$
$=$

SharkCoders

G.S.,

$x(\text{I.F.}) = \int Q(\text{I.F.})\, dy + c$

$x(e^{\tan^{-1}y}) = \int \left(\frac{\tan^{-1}y}{1+y^2}\right)(e^{\tan^{-1}y})\, dy + c$ —— ①

$= I$

Let $t = \tan^{-1}y$

$\frac{1}{1+y^2}\, dy = dt$

$I = \int t\, dt \cdot e^t$

$= \int t\, e^t\, dt$

$= te^t - e^t$

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$I = \tan^{-1}y\, e^{\tan^{-1}y} - e^{\tan^{-1}y}$ —— ②

∴

$x e^{\tan^{-1}y} = \tan^{-1}y \cdot e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$

$\cancel{x = \tan^{-1}y - 1 +}$

2/2/24

1. Exact Differential Eqⁿ
2. Reducible to Exact Differential Eqⁿ
3. Linear Differential Equation
4. Newton's Law of Cooling
5. Electric ~~Current~~ Circuit

. $\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]dx+\frac{2xy}{x^2+y^2}dy=0$

Ans. $\left[\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}\right]dx+\frac{2xy}{x^2+y^2}dy=0$

Comparing with $Mx+Ny=0$,

$$M=\log(x^2+y^2)+\frac{2x^2}{x^2+y^2}$$

$$N=\frac{2xy}{x^2+y^2}$$

$\frac{u}{v}$. $u'v'$ -

$$\frac{\partial M}{\partial y}=\frac{1(2y)}{x^2+y^2}+\frac{2x^2\cdot 0-(2y)(2x^2)}{(x^2+y^2)^2}$$

$$=\frac{2y}{x^2+y^2}-\frac{4x^2y}{(x^2+y^2)^2}=\frac{2x^2y+2y^3-4x^2y}{(x^2+y^2)^2}=\frac{2y^3-2x^2y}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x}=\frac{2y(x^2+y^2)-(2x)(2xy)}{(x^2+y^2)^2}$$

$$=2x^2y+2y^3-4x^2y$$

$$=\frac{2y^3-2x^2y}{(x^2+y^2)^2}.$$

$$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\therefore$ D.E. is exact

2. $[1+e^{x/y}]dx + e^{x/y}[1-x/y]\,dy = 0$

Ans. $[1+e^{x/y}]dx + e^{x/y}[1-x/y]\,dy = 0$

Comparing with $Mdx + Ndy = 0$,

$M = 1 + e^{x/y}$

$N = e^{x/y}[1 - x/y]$

$$\frac{\partial M}{\partial y} = e^{x/y}\left(\frac{0-x}{y^2}\right) = \frac{-xe^{x/y}}{y^2}$$

$$\frac{\partial N}{\partial x} = e^{x/y}\left(-\left(\frac{y-0}{y^2}\right)\right) + xe^{x/y} = e^{x/y}\left(\frac{x}{y^2}\right)$$

$$= e^{x/y}\left(\frac{-1}{y}\right) + e^{x/y}\left(\frac{-x}{y^2}\right)\left(1-\frac{x}{y}\right)$$

$$= \frac{-e^{x/y}}{y} - \frac{xe^{x/y}}{y^2}\left(1-\frac{x}{y}\right)$$

$$= -\frac{e^{x/y}}{y} - \frac{xe^{x/y}}{y^2} + \frac{x^2e^{x/y}}{y^3}$$

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$$= \frac{x^2e^{x/y}}{y^3} - \frac{xe^{x/y}}{y^2} - \frac{e^{x/y}}{y}$$

$$= \frac{e^{x/y}}{y}\left(\frac{x^2}{y^2} - \frac{x}{y} - 1\right)$$

∴ D.E. is not exact.

$$\text{I.F.} = \frac{1}{Mx - Ny}$$

$$Mx - Ny = (1+e^{x/y})x - [e^{x/y}(1-\tfrac{x}{y})]y$$

$$= x + xe^{x/y} - \left(ye^{x/y}\left(1-\frac{x}{y}\right)\right)$$

$$= x + xe^{x/y} - ye^{x/y} + xe^{x/y}$$

$$= 2xe^{x/y} - ye^{x/y} + x$$

2/2/24

$$\int uv\,dx = u\int v\,dx - \int\left[\left(\frac{du}{dx}\right)\int v\,dx\right]dx.$$

$$\int \log(x^2+y^2)\cdot(1)\,dx + \int \frac{2x^2}{x^2+y^2}\,dx = c.$$

$$\log(x^2+y^2)\cdot x - \int \frac{1}{x^2+y^2}(2x)\cdot x\,dx + 2\int \frac{x^2}{x^2+y^2}\,dx = c$$

$$x\log(x^2+y^2) - \int \frac{2x^2}{x^2+y^2}\,dx + \int \frac{2x^2}{x^2+y^2}\,dx = c$$

$$x\log(x^2+y^2) = c$$

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$$\int (1+e^{x/y})\,dx = c \qquad \int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$x + \frac{e^{x/y}}{1/y} = c \qquad M = (1+e^{x/y})$$

$$N = e^{x/y} - \frac{x e^{x/y}}{y}$$

$$x + ye^{x/y} = c$$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y}\left(\frac{-x}{y^2}\right)$$

$$\frac{dy}{dx} = \frac{y}{2y\log y + y - x}.$$

$$M = -y \qquad N = -2y\log y - y + x$$

2/10/24

1. $\cos x \cdot \frac{dy}{dx} + y \sin x = \sec^2 x$

$\cos x \, dy + (y \sin x) dx = \sec^2 x \, dx$

$\cos x \, dy + (y \sin x - \sec^2 x) dx = 0$

Comparing with $M dx + N dy = 0$

$M = y \sin x - \sec^2 x$

$N = \cos x$

$\frac{\partial M}{\partial y} = y\cos x + \sin x - 0 \qquad \frac{\partial N}{\partial x} = -\sin x$

$\frac{\partial M}{\partial y} \neq \frac{\partial M}{\partial x}$

$\therefore$ D.E. is not exact but it is

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$\cancel{Mx - Ny = (y \sin x - \sec^2 x)x - (\cos x)y}$

$\cancel{= xy \sin x - x\sec^2 x - y\cos x}$

$\cancel{= \frac{yx \sin x}{\cos}}$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = y \cos x + \sin x + \sin x$

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Q. $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$

$\frac{dy}{dx} + \frac{y \cancel{\cos} \sin x}{\cos x} = \frac{\sec^2 x}{\cos x}$

$\frac{dy}{dx} + y \tan x = \sec^3 x$

Comparing $\frac{\partial y}{\partial x} + Py = Q$

$P = \tan x$

$Q = \sec^3 x$

I.F.

$= e^{\int P dx}$

$= e^{\int \tan x \, dx}$

$= e^{\log|\sec x|}$

$= \sec x$

$y(I.F) = \int Q (I.F.) dx$

$y(\sec x) = \int \sec^3 x (\sec x) dx$

$= \int \sec^2 (1 + \tan^2 x) dx + c$

$= \int (1 + \tan^2 x) dt + c$

$= \int (1 + t^2) dt + c$

$= \int t + \frac{t^3}{3} + c$

G.S.:

$y(I.F.) = \int Q (I.F.) dx + c$

$y(\sec x) = \int \sec^3 x (\sec x) dx$

$\cancel{y \sec x} \quad \frac{\cancel{\sec^5 x}}{5} = \frac{\cancel{y \sec^6 x}}{5} \quad y \sec x = \frac{\sec^5 x}{5}$

$y = \frac{\sec^4 x}{5}$

$G.S. = \int Q (I.F.) dx + c = y(I.F.)$

$y \sec x = \int \sec^2 x \, dx$

$y \sec x = \cancel{\frac{\sec^3 x}{3}} \tan x$

$y = \sec^2 x = \frac{1}{1 + x^2}$

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2/2/24

2. $\frac{dy}{dx} + \frac{y}{1-x} = x(x-1)$

Ans $\frac{dy}{dx} + \left(\frac{1}{1-x}\right)y = x^2 - x$

Comparing with $\frac{dy}{dx} + Py = Q$,

$P = \frac{1}{1-x}$, $Q = x^2 - x$

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\cancel{\log(1-x)} - \log(1-x)}$

$= e^{\log(1-x)^{-1}}$

$= \frac{1}{1-x}$

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G.S. :-

$y(\text{I.F.}) = \int Q(\text{I.F.})\,dx$

$y\left(\frac{1}{1-x}\right) = \int \cancel{(x^2-x)} \frac{x(x-1)}{-(x-1)}\,dx$

$\frac{y}{1-x} = \int -x\,dx$

$\frac{y}{1-x} = \frac{-x^2}{2}$

$y = \frac{-x^2 + x^3}{2}$

$2y = \frac{x^3 - x^2}{\cancel{2}}$

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Q. $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = (1-\sqrt{x})$

Ans. $\frac{dy}{dx} + \frac{y}{\sqrt{x}(1-x)} = (1-\sqrt{x})$ —— (1)

Comparing with $\frac{dy}{dx} + py = Q$,

$p = \frac{1}{\sqrt{x}(1-x)}$ $\qquad Q = 1-\sqrt{x}$

$x^{-1/2} - x^{3/2}$ $\quad \sqrt{x}$

$$\text{I.F.} = e^{\int p\,dx}$$
$$= e^{\int \frac{1}{\sqrt{x}(1-x)} dx}$$
$$= e^{\log[\sqrt{x}(1-x)]}$$
$$= e^{\log[\sqrt{x}(1-x)]}$$

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$\text{I.F.}\ e^{\int \frac{1}{(1-x)\sqrt{x}} dx} = e^{I}$

$I = \int \frac{1}{\quad}$

Q. $\cos x \frac{dy}{dx} + y = \sin x$

$\frac{dy}{dx} + \frac{y}{\cos x} + \tan x$

$p = \frac{1}{\cos x} = \sec x \qquad Q = \tan x.$

$\text{I.F.} = e^{\int \sec x\,dx} = e^{\int \sec x + \tan x\,dx} = e^{\log(\sec x + \tan x)}$

$= \sec x + \tan x$

2/2/24

G.S. =

$$y(\text{I.F.}) = \int Q(\text{I.F.})\,dx$$

$$y(\sec x + \tan x) = \int \tan x \,\cancel{d}\,(\sec x + \tan x)\,dx$$

$$y\sec x + y\tan x = \cancel{\tan x \sec x + \tan^2 x}$$

$$y(\sec x + \tan x) = \int (\sec x + t)\,dt$$

$$= t.\sec x + \frac{t^2}{2}$$

$$y(\sec x + \tan x) = \sec x \tan x + \frac{\tan^2 x}{2}$$

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Newton's Law of Cooling

The temperature of body changes at the rate ~~of~~ which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

∴ if θ_0 is the temperature of the surrounding medium and θ that of the body at any time t, then.

$\frac{d\theta}{dt} \propto \theta - \theta_0$ where k is constant.

$$\boxed{\frac{d\theta}{dt} = -k(\theta - \theta_0)}$$

Q. • a metal wall is heated to a temperature of $100°C$ and at time $t = 0$. It is placed in ~~order~~ water which is mentioned at $40°C$, if the temp of the wall is reduced to $60°C$ in 4 mins, find the time at which the temp of the wall is $50°C$.

Ans.

$\theta_0 = 40$

$\theta_1 = 100°$

$t_1 = 0s$

$\theta_2 = 60°$

$t = 4$ mins

$\theta_3 = 50°$

By Newton's Law of Cooling,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \quad \text{~~where } \theta_0\text{~~}:$$

$$\frac{d\theta}{dt} = -k(\theta - 40)$$

$$\frac{d\theta}{\theta - 40} = -k\,dt$$

∴ This is variable seperable form.

Integrating both sides,

$$\int \frac{d\theta}{\theta - 40} = \int -k\,dt$$

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$$\int_{100}^{60} \frac{d\theta}{\theta - 40} = \int_{0}^{4} -k \, dt$$

$$\therefore \left[\log(\theta - 40)\right]_{100}^{60} = -k\left[t\right]_0^4$$

$$\log_e(20) - \log_e(60) = -4k + 0$$

$$\log_e\left(\frac{20}{60}\right) = -4k$$

$$\ln\left(\frac{1}{3}\right) = -4k$$

$$k = \frac{1}{4}\log 3$$

Integrating eq ①,

$$\int_{100}^{50} \frac{d\theta}{\theta - 40} = \int_{0}^{t} -k \, dt$$

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$$\left[\log(\theta - 40)\right]_{100}^{50} = -k\left[t\right]_0^t$$

$$\log 10 - \log 60 = -kt$$

$$\log\left(\frac{1}{6}\right) = -kt$$

$$-\log 6 = -kt$$

$$k = \frac{\log 6}{t}$$

$$\frac{1}{4}\log 3 = \frac{\log 6}{t}$$

$$t = \frac{4\log 6}{\log 3}$$

$$= 6.5 \text{ mins.}$$

Q. According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temp of the air is 300°C, and the substance cooled from cooled from 370°C to 340°C in 15 mins. Find when the temp will be 310°C.

Ans. $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

$\theta_0 = 300°$ 370 → 0

370   340 → 15

310 → t

$$\int_{370}^{340} \frac{d\theta}{\theta - 300} = \int_0^{15} -k\, dt$$

$$[\log(\theta - 300)]_{370}^{340} = -k[t]_0^{15}$$

$$-\log 70 + \log 40 = -15k$$

$$-\log \frac{7}{4} = -15k$$

$$\log\left(\frac{4}{7}\right) = -15k$$

$$\log\left(\frac{7}{4}\right) = 15k$$

$$k = \frac{1}{15} \log\left(\frac{7}{4}\right)$$

Integrating eq ①.

$$\int_{370}^{310} \frac{d\theta}{\theta - 300} = \int_0^t -k\, dt$$

$$[\log(\theta - 300)]_{370}^{310} = -k[t]_0^t$$

$$\log 10 - \log 70 = -kt$$

$$\log\left(\frac{1}{7}\right) = -kt$$

$$\log 7 = kt$$

$$k = \frac{\log 7}{t}$$

$$\frac{1}{15} \log\left(\frac{7}{4}\right) = \frac{\log 7}{t}$$

$$t = \frac{15 \log 7}{\log(7/4)}$$

$t = 52.16$ mins.

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3/2/24

Q. A body at temp 100°C placed in a room whose temp is 20°C and cooled to 60° in 5 mins. What will be the temp of body after 8 mins.

Ans. $\theta_0 = 20°C$.

$100 \to 0$
$60 \to 5$
$\theta \to 8$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\frac{d\theta}{\theta - \theta_0} = -k\,dt$$

$$\int_{100}^{60} \frac{d\theta}{\theta - 20} = \int_0^5 -k\,dt$$

$$[\log(\theta - 20)]_{100}^{60} = -k[t]_0^5$$

$$\log(40) - \log(80) = -5k$$

$$\log\left(\frac{1}{2}\right) = -5k$$

$$\log 2 = 5k$$

$$k = \frac{\log 2}{5}$$

$$\int_0^{\theta} \frac{d\theta}{\theta - 20} = \int_0^8 -k\,dt$$

$$[\log(\theta - 20)]_{100}^{\theta} = -k[t]_0^8$$

$$\log(\theta - 20) - \log(80) = -8k$$

$$\log\left(\frac{\theta - 20}{80}\right) = -8k$$

$$\log\left(\frac{\theta - 20}{80}\right) = \frac{-8\log 2}{5}$$

$$\log\left(\frac{\theta}{80} - \frac{1}{4}\right) = \frac{-8\log 2}{5}$$

$$\log\left(\frac{80}{\theta - 20}\right) = \frac{8\log 2}{5}$$

$$\log\left(\frac{80}{\theta - 20}\right) = \log 2^{8/5}$$

$$\frac{80}{\theta - 20} = 2^{8/5} = 3.03(\theta - 20)$$

$$80 = 3.03\theta - 60.6$$

$$140.6 = 3.03\theta$$

$$46.4 = \theta$$

3/2/24

Q. A body originally having 80°c cooled down to 60°C in 20 mins. The temp of the air being 40°c, what will be the temp of body after 40 mins. from the original ?

$\boxed{\text{Ans} = 50°C.}$

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Simple Electric Circuit

Circuit is made up of :

1. 3 passive elements
2. An active element

~~3~~.

Basic Relation

1. $i = \frac{dq}{dt}$ OR $q = \int i \, dt$

Rate of flow of electricity /charge

2. Voltage drop across resistance $R = Ri$ (Ohm's Law)

3. $L = L\frac{di}{dt}$

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4. Voltag drop across capacitance ~~Q=Q~~ C ($C = q/L$)

5. Kirchoff's Law
The algebraic sum of ~~vo~~ltage drop around any closed circuit is equal to ~~a~~ resultant electromotive force in the same circuit.

7. The algebraic sum of the current, flowing into a node $= 0$.

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D.E. of LR circuit

$$L\frac{di}{dt} + Ri = E \quad (E = e.m.f.)$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{E}{L}$$

D.E. of CR circuit

$$Ri + \frac{q}{C} = E$$

$$R\frac{dq}{dt} + \frac{q}{C} = E$$

$$\frac{dq}{dt} + \left(\frac{1}{RC}\right)q = \frac{E}{R}$$

Q. A resistance of 100 Ω and L of 0.5 henry are connected in series with a battery of 20V. Find the current in a circuit as a function of t.

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Ans. $R = 100$

$L = 0.5$

$V = 20$

$$L\frac{di}{dt} + Ri = E$$

$$\Rightarrow \frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{E}{L}$$

which is linear D.E. in i.

$$P = \frac{R}{L} \qquad q = \frac{E}{L}$$

$$\boxed{y(I.F.) = \int Q(I.F.)\,dx + c}$$

$$I.F. = e^{\int P\,dt}$$

$$= e^{\int \frac{R}{L}dt}$$

$$= e^{\frac{Rt}{L}}$$

G.S. =

$$i\left(e^{\frac{Rt}{L}}\right) = \int \frac{E}{L} \cdot e^{\frac{Rt}{L}} \cdot dt + A$$

$$\because \int e^{ax}\, dx = \frac{e^{ax}}{a}$$

$$i\left(e^{\frac{Rt}{L}}\right) = \frac{E}{L} \frac{e^{\frac{Rt}{L}} (L)}{(R)}$$

$$i\left(e^{\frac{Rt}{L}}\right) = \frac{E e^{\frac{Rt}{L}}}{R} + A$$

$$i = \frac{E}{R} + A e^{-\frac{Rt}{L}} \quad \text{--- (1)}$$

But $i = 0$ at $t = 0$

$$0 = \frac{E}{R} + A$$

$$A = -\frac{E}{R} \quad \text{--- (2)}$$

Substituting (2) in (1),

$$i = \frac{E}{R} - \frac{E}{R} e^{-\frac{Rt}{L}}$$

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$i = \frac{20}{100}\left(1 - e^{-\frac{100 \times t}{0.5}}\right)$$

$$= \frac{1}{5}\left(1 - e^{-200t}\right)$$

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Q. A resistance of 150 ohm, and an inductance of 0.3 henry are connected in series with a battery of 25V. Find the current in the circuit as a funcⁿ of t.

Ans. $\cancel{L\frac{di}{dt} + Ri = E}$

$\cancel{\frac{di}{dt} + \left(\frac{R}{L}\right)}$

Q. In a circuit, a constant electromotive force of E volts, is applied to a circuit containing a constant resistance R Ω in a series and a constant L henry if the initial current is 0, show that the current builds up to half its theoretical maximum in L/R log 2 secs.

Q.

$$\boxed{\begin{array}{l} \text{At } t \to \infty, \\ I_{max} = \frac{E}{R} \end{array}}$$

$$\frac{I_{max}}{2} = \frac{E}{2R}$$

$$\frac{1}{2}\cdot\frac{E}{R} = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$-\frac{Rt}{L} = \log(1/2) = -\log 2.$$

$$\frac{Rt}{L} = \log 2$$

$$t = \frac{L}{R}\log 2 \text{ sec}$$

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Q. In a circuit containing inductance L, resistance and voltage V is given by $E = RI + L\frac{dI}{dt}$

Given $L = 640$ henry

$R = 250\ \Omega$

$E = 500\ V$

I being 0 when $t = 0$, find the ~~type~~ time that elapses before it reaches 80% of its max value.

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MIND MAP

INTEGRAL CALCULUS

Reduction formula and properties

Gamma func^n

Beta func^n

Reduction formula and other properties

1. Reduction formula
   (I_n = I_{n-1} etc. and I_{n-2})

2. $$\int_0^{\pi/2} \sin^n x \, dx = \left[\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\cdots\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\right](1) \;;\; n \text{ is odd.}$$

$$= \left[\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\cdots\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\right]\left(\frac{\pi}{2}\right) \;;\; n \text{ is even}$$

3. $$\int_0^{\pi/2} \cos^n x \, dx = \left[\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\cdots\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1)\right](1) \;;\; n \text{ is odd}$$

$$= \left[\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\cdots\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1)\right]\frac{\pi}{2} \;;\; n \text{ is even}$$

4. $$\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$$

$$= \left\{\frac{[(m-1)(m-3)(m-5)\ldots 2 \text{ or } 1][(n-1)(n-3)(n-5)\ldots 2 \text{ or } 1]}{(m+n)(m+n-2)(m+n-4)\ldots 2 \text{ or } 1}\right\} Q$$

where $Q = \pi/2$ if m, n are even

$Q = 1$ if otherwise

5. $\int_0^{\pi} \sin^n x \, dx = 2\int_0^{\pi/2} \sin^n x \, dx$

6. $\int_0^{\pi} \cos^n x \, dx = 2\int_0^{\pi/2} \cos^n x \, dx$; n is even

$= 0$; n is odd

7. $\int_0^{2\pi} \sin^n x \, dx = 4\int_0^{\pi/2} \sin^n x \, dx$; n is even

$= 0$; n is odd

8. $\int_0^{2\pi} \cos^n x \, dx = 4\int_0^{\pi/2} \cos^n x \, dx$; n is even

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$= 0$; n is odd

9. $\int_0^{\pi} \sin^m x \cos^n x \, dx = 2\int_0^{\pi/2} \sin^m x \cos^n x \, dx$; n is even

$= 0$; n is odd

10. $\int_0^{2\pi} \sin^m x \cos^n x \, dx = 4\int_0^{\pi/2} \sin^m x \cos^n x \, dx$; m, n is even

$= 0$; otherwise

11. $\int_{-a}^{a} f(x)\, dx = 2\int_0^{a} f(x)\, dx$; $f(x)$ is even

$= 0$; $f(x)$ is odd.

12. Even func^n : $f(x) = f(-x)$
Odd func^n : $f(x) = -f(-x)$

13. $\sin x \rightarrow$ odd $\qquad \cos x \rightarrow$ even

$\sin^{2n} x \rightarrow$ even $\qquad \cos^{n} x \rightarrow$ even

$\sin^{2n+1} x \rightarrow$ odd

14. Even = E $\quad$ Odd = 0

$(E)(E) = E \qquad (E)(O) = 0$

$(O)(O) = E \qquad (O)(E) = 0$

15. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\boxed{\text{Gamma Function}}$

The func^n of n defined by the integral,

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$$I = \int_0^\infty e^{-x} x^{n} dx = \Gamma(n+1) \quad ; n > 0$$

Properties:

1. $\Gamma 1 = \Gamma 1$
2. $\Gamma 0 = \infty$
3. $\Gamma(n+1) = n\Gamma n$
4. $\Gamma(n+1) = n!$
5. $\Gamma(1/2) = \sqrt{\pi}$

TYPES

- Reduction of integral
- $\int_0^\infty \frac{x^a}{a^x} dx$
- $\int_0^\infty a^{bx^n} dx$
- log func^n's

$\boxed{\text{NOTE:- For } \log x \text{, we put } \log x = -y. \text{ For } \log(1/x) \text{, we put } \log(1/x) = y.}$

Beta Function

The func^n of m and n defined by integral.

$$I = \int_0^1 x^{m} (1-x)^{n} \, dx = 0 \quad ; \quad m > 0, \; n > 0$$

$$= \beta(m+1, n+1)$$

Properties:

1. $\beta(m, n) = \beta(n, m)$
2. $\beta(m, n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma (m+n)}$
3. $\displaystyle\int_0^{\pi/2} (\sin^p \theta \cdot \cos^q \theta) \, d\theta = \frac{1}{2} \beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$
4. $\Gamma p \, \Gamma (1-p) = \dfrac{\pi}{\sin p\pi}$ $[0 < p < 1]$

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Integral Calculus.

Reduction Formula

Let $I_n = \int f(n, x)\, dx$ where n is a parameter,

If the value of I_n can be expressed in terms of I_{n-1} or I_{n-2} that is in terms of lower index then this expression. ~~is called~~.

Reduction formulae are useful to integrate some special kind of integration in very simple way.

Q. Find integration of $\int \sin^n x\, dx$ where $n \geq 2$ (n...
Also find $\int_0^{\pi/2} \sin^n x\, dx$.

Ans. Let $I_n = \int \sin^n x\, dx$

as $\sin^n x$ is w/r to (n, x)

$$= \int \sin^{n-1}(x) \cdot \sin x\, dx$$

Using Integration by parts,

$$I = \sin^{n-1}(x) \cdot (-\cos x) - \int [(n-1)\sin^{n-2}x \cdot \cos(x)][-\cos(x)]\, dx$$

$$= -\sin^{n-1}(x)\cos x + \int (n-1)\sin^{n-2}x \cdot \cos^2 x\, dx$$

$$= -\sin^{n-1}x \cos x + \int (n-1)\sin^{n-2}x\,(1 - \sin^2 x)\, dx$$

$$= -\sin^{n-1}x \cdot \cos x + \int (n-1)\sin^{n-2}x\, dx \int (n-1)\sin^n x\, dx$$

$$= -\sin^{n-1}x \cos x + (n-1)\int \sin^{n-2}x\, dx - (n-1)\int \sin^n x\, dx$$

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$I_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2} - (n-1) I_n$

We know that,

$I_n = \int \sin^n x \, dx$

$I_{n-2} = \int \sin^{n-2} x \, dx$

$[1 + (n-1)] I_n = -\sin^{n-1}(x) \cos x + (n-1) I_{n-2}$

$n I_n = -\sin^{n-1}(x) \cos x + (n-1) I_{n-2}$

$I_n = \frac{1}{n} \sin^{-1} x \cdot \cos x + \left(\frac{n-1}{n}\right) I_{n-2}$

$\therefore \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \left(\frac{n-1}{n}\right) \int \sin^{n-2}(x) \, dx$

$\Rightarrow \int_0^{\pi/2} \sin^n x \, dx = \left[-\frac{1}{n} \sin^{n-1}(x) \cos x\right]_0^{\pi/2} + \left(\frac{n-1}{n}\right) \int_0^{\pi/2} \sin^{n-2}(x) dx$

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$= \left(\frac{n-1}{n}\right) \int_0^{\pi/2} \sin^{n-2}(x) \, dx$

$\therefore I_n = \left(\frac{n-1}{n}\right) I_{n-2}$

$\int_0^{\pi/2} \sin^n(x) \, dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right),$

if n is even

$\int_0^{\pi/2} \sin^n(x) \, dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \cdots \left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1)(1),$

if n is odd.

Q. $\int_0^{\pi/2} \sin^7 x \, dx$

Ans. $I = \int_0^{\pi/2} \sin^7 x \, dx$

$f(x) = \sin^7 x$

7 is odd.

$$I = \left[\cancel{(6\cdot4\cdot2)} \; \left(\frac{6}{7}\right)\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1) \right](1)$$

$$= \frac{16}{35}.$$

Q. $\int_0^{\pi/2} \sin^4 x \, dx$

Ans. $I = \int_0^{\pi/2} \sin^4 x \, dx$

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$f(x) = \sin^4 x$

4 is even.

$$I = \left[\cancel{(3\cdot1)} \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) \right]$$

$$= \left(\frac{3\pi}{16}\right)$$

8/2/24

1. $$\int_0^{\pi/2} \sin^n x \, dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) \text{ if } n \text{ is even}$$

2. $$\int_0^{\pi/2} \cos^n x \, dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\cdots\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1) \text{ if } n \text{ is odd.}$$

3. $$\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx =$$

$$\left[\frac{[(m-1)(m-3)(m-5)\ldots 2 \text{ or } 1][(n-1)(n-3)(n-5)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)(m+n-4)\ldots 2 \text{ or } 1]}\right] \times Q$$

where $Q = \frac{\pi}{2}$; if m, n are even

$Q = 1$; otherwise

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Q. Evaluate $\int_0^{\pi/2} \sin^8 x \, dx$.

Ans. Here $n = 8$

$$\therefore \int_0^{\pi/2} \sin^8 x \, dx = \left(\frac{7}{8}\right)\left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)$$

$$= \frac{105\pi}{768} = \frac{35\pi}{256}$$

Q. Evaluate $\int_0^{\pi/2} \cos^7 x \, dx$

Ans. Here $n = 7$ is odd.

$$\therefore \int_0^{\pi/2} \cos^7 x \, dx = \left(\frac{6}{7}\right)\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1) = \frac{16}{35}$$

Q. Evaluate $\int_0^{\pi/2} \sin^8 x \cdot \cos^6 x \, dx$

Ans. Here $m=8$, $n=6$ are even

$$\int_0^{\pi/2} \sin^8 x \cdot \cos^6 x \, dx = \frac{(7\cdot5\cdot3\cdot1)(5\cdot3\cdot1)}{14\cdot12\cdot10\cdot8\cdot6\cdot4\cdot2}\cdot\frac{\pi}{2}$$

$$= \frac{5\pi}{4096}$$

Q. Evaluate $\int_0^{\pi/2} \sin^7 x \cdot \cos^6 x \, dx$

Ans.

$$\int_0^{\pi/2} \sin^7 x \cdot \cos^6 dx =$$

$$\cancel{(6\cdot5\cdot3\cdot2}$$

$$\cancel{\frac{(6\cdot5\cdot4\cdot3\cdot2\cdot1)(6\cdot5\cdot4\cdot3\cdot2\cdot1)(1)}{(13)(11)(9)(7)(5)(3)(1)}} = \frac{3840}{1001}$$

$$\frac{(6\cdot4\cdot2)\cdot(5\cdot3\cdot1)(1)}{(13)(11)(9)(7)(5)(3)(1)} = \frac{16}{3003}$$

☆ Formulae

1. $\int_0^{\pi} \sin^n x \, dx = 2\int_0^{\pi/2} \sin^n x \, dx$ for all 'n'

2. $\int_0^{\pi} \cos^n x \, dx = 2\int_0^{\pi/2} \cos^n x \, dx$, if n is even integer

~~if n is odd integer~~

$= 0$ if n is odd integer

3. $\int_0^{2\pi} \sin^n x \, dx = 4\int_0^{\pi/2} \sin^n x \, dx$, if n is even integer

~~otherwise odd integer~~

$= 0$ if n is odd integer

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4. $\int_0^{2\pi} \cos^n x \, dx = 4\int_0^{\pi/2} \cos^n x \, dx$, if n is even integer.

$= 0$, if n is odd integer

5. $\int_0^{\pi} \sin^m x \cdot \cos^n x \, dx = 2\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$, if n is even integer

$= 0$, if n is odd integer

6. $\int_0^{2\pi} \sin^m x \cdot \cos^n x \, dx = 4\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx$, if m, n both are even.

$= 0$, otherwise.

Q. Evaluate $\int_0^{\pi/6} \sin^5 3x \, dx$.

Ans.

put $3x = \theta$

$3dx = d\theta$

$dx = \frac{1d\theta}{3}$

x	0	$\pi/6$
θ	0	$\pi/2$

$I = \int_0^{\pi/6} \sin^5 3x \, dx = \int_0^{\pi/2} \sin^5\theta \cdot \frac{d\theta}{3}$

$= \frac{1}{3}\int_0^{\pi/6} \sin^5\theta \, d\theta$

$= \frac{1}{3}\left[\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1)(1)\right]$

$I = \frac{8}{45}$

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Q. $\int_0^{\pi/4} \cos^8 2x \, dx$

Ans. Let $2x = \theta$

$2\,dx = d\theta$

$dx = \frac{d\theta}{2}$ —— (1)

$\int_0^{\pi/4} \cos^8 2x \, dx = \int_0^{\pi/4} \cos^8\theta \frac{d\theta}{2}$ [From (1)]

$\frac{1}{2}\int_0^{\pi/4} \cos^8\theta \, d\theta =$ [By reduction form]

$\frac{1}{2}\left[\left(\frac{7}{8}\right)\left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)\right] =$

$\left[\frac{5\pi}{128}\right]$

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Q. $\int_0^{2a} x\sqrt{2ax - x^2}\, dx$

Ans. Let $I = \int_0^{2a} x\sqrt{2ax - x^2}\, dx$

$x = 2a\sin^2\theta$

$dx = \cancel{2a(2\sin\theta\cos\theta)}\ 2a(2\sin\theta)$

$dx = 2a(2\sin\theta \cdot \cos\theta)$

x	0	$2a$
θ	0	$\pi/2$

$I = \int_0^{\pi/2} 2a\sin^2\theta \left(\sqrt{2a(2a\sin^2\theta) - 4a^2\sin^4\theta}\right)(4a\sin\theta\cos\theta\, d\theta)$

$= 8a^2\int_0^{\pi/2} \sin^3\theta\sqrt{4a^2\sin^2\theta - 4a^2\sin^4\theta} \cdot \cos\theta \, d\theta$

$$= 8a^2 \int_0^{\pi/2} \sin^3\theta \cos\theta \sqrt{4a^2\sin^2\theta(1-\sin^2\theta)}\, d\theta$$

$$= 8a^2 \int_0^{\pi/2} \sin^3\theta \cos\theta \,(2a \sin\theta \cos\theta)\, d\theta$$

$$= 16a^3 \int_0^{\pi/2} \sin^4\theta \cos^2\theta \, d\theta$$

$$= 16a^3 \left[\frac{(3\cdot1)(1)}{6\cdot4\cdot2}\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{a^3}{2}\pi$$

Q. $\int_0^{\pi/8} \sin^6 4x \, dx$

Ans. $I = \int_0^{\pi/8} \sin^6 4x \, dx$

Let $4x = \theta$

$4\, dx = d\theta$

$dx = \frac{d\theta}{4}$

if $x = \pi/8$,

$\cancel{dx = \frac{\pi}{8}}$

$\frac{\pi}{8} = \frac{d\theta}{4}$

$\frac{\pi}{2} = d\theta$

$I = \int_0^{\pi/2} \sin^6\theta \, d\theta \left(\frac{1}{4}\right).$

$I = \frac{1}{4}\left[\left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)\right]$

$I = \frac{15\pi}{64\times 6} = \frac{5\pi}{128}$

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Property:

1. $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \quad ; \text{ if } f(x) \text{ is even func}^n$$
$$= 0 \quad , \text{ if } f(x) \text{ is odd func}^n$$

2. Even func^n $\Rightarrow f(-x) = f(x)$.

Odd func^n $\Rightarrow f(-x) = -f(x)$

3. $\sin x$ is odd. $(\sin(-x) = -\sin x)$

4. $\cos x$ is even $(\cos(-x) = \cos x)$

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5. $\sin^{2n} x$, is even.

6. $\sin^{2n+1} x$ is odd

7. $\cos^n x$ is even.

8. E = even

O = odd

$\therefore$ (E)(E) = E (E)(O) = O

(O)(O) = E (O)(E) = O

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Date
Page

Q. $\int_{-\pi/2}^{\pi/2} \cos^5 x \, dx$

Ans. $I = \int_{-\pi/2}^{\pi/2} \cos^5 x \, dx$

Let $\cos^5 x = f(x)$

$\cos^n x$ is ~~odd~~ even,

$\therefore f(x) = \cos^5 x$ is an even func^n.

$$I = 2\int_0^{\pi/2} \cos^5 x \, dx$$

$$= 2\left[\left(\frac{4}{5}\right)\left(\frac{2}{3}\right)(1)(1)\right]$$

$$= \frac{16}{15}$$

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Q. $\int_{-\pi/2}^{\pi/2} \cos^6 x \, dx.$

Ans. $I = \int_{-\pi/2}^{\pi/2} \cos^6 x \, dx$

Let $f(x) = \cos^6 x$.

$\cos^n x$ is an even func^n.

$f(x) = \cos^6 x$ is even.

$$\int_{-\pi/2}^{\pi/2} f(x)\, dx = 2\int_0^{\pi/2} \cos^6 x \, dx.$$

$$= 2\left[\left(\frac{5}{6}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{15\pi}{48} = \frac{5\pi}{16}$$